RELIABILITY ANALYSIS OF DIFFERENT TRANSFORMATION MODELS BETWEEN TERRESTRIAL AND SATELLITE SYSTEMS AS APPLIED TO THE EGYPTIAN GEODETIC NETWORKS

BY

Dr. Ahmed A. Shaker
Ass.Prof. of Geodesy
Faculty of Engineering
Zagazig University

Eng. Mohamed A. Shaban
Faculty of Engineering
Assiut university

يهدف هذا البحث الى دراسة النماذج الرياضيـــــه المختلفـــه المستخدمه لتحويل الاحداثيـــات المأخـــوذه من الاقمـــار الصناعيـــه الى احداثيـات ارضيـه بالاضافـه الى دراسة العوامل المختلفـــة المؤثـــرة على نتائج هذه النماذج الرياضيـه وذلـــــك بغرض الوصـــــول الى استنباط افضـــــل حالـــه لكل نموذج بحيث يمكـــن استخدامه لتحويل الاحداثيـــات المأخـــوذه بالاقمـــــار الصناعيـــــه الى احداثيات ارضيـــــــــه وذلك فى جمهوريـــــة مصـــر العربيـــــــه ٠

ABSTRACT

Single trials has been carried out in Egypt for the combination of Doppler sattelite and terrestrial Egyptian geodetic network using different number of common points. Now, with the increase of the number of observed Doppler stations in Egypt (22 points), it was found reasonable to analyse the reliability of six models of transformations between Doppler and trrestrial systems. Experimental results showd that the most simple and reasonable model to be used in Egypt is Molodensky model.

I-Introduction

Satellite Doppler positioning techniques play an important role in establishing, extending or strengthening of geodetic networks. They also achive a fundamental goal of geodesy through the determination of absolute position with uniform accuracy at all points on the earth's surface. It is well known that the Doppler coordinates are given with respect to a world-wide geocentric coordinate system , while any national geodetic coordinates are usually given with respect to a certian specified local datum. Accordingly before making any combination between the two systems, the transformation parameters between them should be well determined. Currently their are several transformation models in use for such a transformation. All these models differ in both, number and treatment of the transformation parameters, that is translation , rotation and scale. The main objective of this paper is to investigate all possible models to decide upon the best of them as far as the Egyptian situation is concerned.

2-TRANSFORMATION MODELS UNDER CONSIDERATIONS

Two different groups of transformation models are considerd in this paper. The first group includes *BURSA , MOLODENSKY & VIES models*, which contain one set of rotation parameters. The second groupe includes *KRAKIWSKY-THOMSON & HOTINE models*, which contain two sets of rotation parameters.

2-1 MODELS CONTAINING ONE SET OF ROTATION PARAMETERS

A) BURSA MODEL:

In *BURSA MODEL* the relationship between terrestrial and doppler system is expressed by three translations (X_o, Y_o, Z_o), three rotations (E_x ,E_y ,E_z) and scale change (Δ), *fig (1)*. The rotation angles are positive for counter-clockwise rotations about the respective geodetic X ,Y ,Z axis as viewed from the end of the positive axis making a right hand system [1], [2]. The transformation equation between the two systems is given in vector form as;

$$\bar{F} = \bar{T} + (1 + \Delta) R \bar{G} - \bar{D} = 0 \qquad (1)$$

The final form of the transformation equation between the two systems after omitting terms containing the products of scale and rotation is given as;

$$F_k = \bar{T} + \Delta \; \bar{G} + R1 \; \bar{G} + \bar{G} - \bar{D} = 0$$

$$F_k = \begin{bmatrix} Xo \\ Yo \\ Zo \end{bmatrix} + \Delta \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_G + \begin{bmatrix} 0 & Ez & -Ey \\ -Ez & 0 & Ex \\ Ey & -Ex & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_G + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_G - \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_D = 0$$

Where

- T = Translation vector between the origin of the satellite and geodetic systems.
- Δ = Scale change
- R = Product of three rotation matrices about the geodetic axis $[X, Y, Z]^T_G$
- $R1$ = $R - I$
- G = Vector of geodetic coordinates $[X, Y, Z]^T_G$
- D = Vector of Doppler or satellite coordinates $[X, Y, Z]^T_G$

B) MOLODENSKY MODEL

In this model ,*fig(2)* , the position vector of the datum origin or initial point (i) is known relative to the geodetic system which is assumed to be parallel to the satellite system and the involved rotations are considerd to be around the geodetic axis of the initial point [1],[2]. The final form of the transformation equation for any point (k) in this system is given by;

$$F_k = \bar{T} + \Delta \, (\bar{G}_k - \bar{G}_i) + Q \, . \, (\bar{G}_k - \bar{G}_i) + \bar{G}_K - \bar{D}_k = 0 \quad \text{...(2)}$$

$$F_k = \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} + \Delta \begin{bmatrix} X_k - X_i \\ Y_k - Y_i \\ Z_K - Z_I \end{bmatrix}_G + \begin{bmatrix} 0 & W_z & -W_y \\ -W_z & 0 & W_x \\ W_y & -W_x & 0 \end{bmatrix} \begin{bmatrix} X_k - X_i \\ Y_k - Y_i \\ Z_k - Z_i \end{bmatrix}_G + \begin{bmatrix} X_k \\ Y_k \\ Z_k \end{bmatrix}_G - \begin{bmatrix} X_k \\ Y_k \\ Z_k \end{bmatrix}_D = 0$$

C) VEIS MODEL

In this model . *fig (3)*. the rotations denoted by $\delta\alpha$ $\delta\xi$ $\delta\eta$ are in a local geodetic system at the initial point (i), and its U axis is tangent to the geodetic meridian with positive direction toward north; the V axis is perpendicular to the meridian plane and it is positive eastward. Finally, the W axis is positive towards the outside extension of the direction of the geodetic normal. The above rotations $\delta\eta$ $\delta\xi$ $\delta\alpha$ denote positive rotations about U,V,W axes respectively, and ϕ_i ,λ_i & h_i are the geodetic curvilinear coordinates of the initial point. Similar to equation (2) one obtains;

$$\mathbf{F} = \mathbf{T} + \overline{\mathbf{G}}_i + (1 + \Delta)\ \mathbf{M}\ (\overline{\mathbf{G}}_k - \overline{\mathbf{G}}_i) - \overline{\mathbf{D}}_k = 0 \qquad ..(3)$$

the rotation matrix M ,[1] is given as

$$M = R_3(180 - \lambda_i)\ R_2(90 - \phi_i)\ R(\delta\eta, \delta\xi, \delta\alpha)\ R_2(\phi_i - 90)\ R_3(\lambda_i - 180)$$

$$\mathbf{M} = \begin{bmatrix} 1 & -\delta\alpha \sin\phi_i - \delta\eta \cos\phi_i & +\delta\alpha \cos\phi_i \sin\lambda_i + \delta\xi \cos\lambda_i - \delta\eta \sin\phi_i \sin\lambda_i \\ -\delta\alpha \sin\phi_i + \delta\eta \cos\phi_i & 1 & -\delta\alpha \cos\phi_i \cos\lambda_i + \delta\xi \sin\lambda + \delta\eta \sin\phi_i \cos\lambda_i \\ -\delta\alpha \cos\phi_i \sin\lambda_i + \delta\xi \cos\lambda_i + \delta\eta \sin\phi_i \sin\lambda_i & +\delta\alpha \cos\phi_i \cos\lambda_i - \delta\xi \sin\lambda_i - \delta\eta \sin\phi_i \cos\lambda_i & 1 \end{bmatrix}$$

Thus the only difference between *MOLODENSKY & VEIS models* is that the rotations in the *VEIS* model are the well known deflection components representing the misorientation of the terrestrial network at the initial point.

2-2 MODELS CONTAINING TWO SETS OF TRANSFORMATION PARAMETERS

Some of the models included in the first group assumed that the geodetic coordinate system is parallel to the satellite coordinate system,this is generally not true and the rotation parameters should be applied to deal with the lack of parallism between the two systems. The second group of models solved these problem by applying two sets of rotation parameters. One of them, with translation parameters, is used to model the datum position and orientation relative to the satellite system. The other set of rotations with scale factor is used to deal with the scale and orientation distortions in the geodetic network relative to the initial point. These models are *KRAKIWSKY-THOMSON & HOTINE MODELS.*

A) KRAKIWSKY-THOMSON MODEL.

This model, *fig(4)*, can be represented as follows;

$$\mathbf{F}_k = \bar{\mathbf{T}} + \mathbf{R}\,\bar{\mathbf{G}}_i + (1+\Delta)\,\mathbf{M}\,\bar{\mathbf{G}}_{ik} - \bar{\mathbf{D}}_k = 0 \qquad \ldots(4)$$

This model contains ten unkown parameters, Three translation parameters ,scale change and two sets of rotation parameters. The first set (**R**), is for the misorientation of the geodetic system with respect to the satellite system, and a second set of rotations (**M**) is for the misoriented terrestrial network.

In the above model we have two sets of unknown rotations, accordingly a special treatment needs to be applied to solve such a model. This is achived by spliting up the common points into two zones, inner and outer zone.

The reason for this spliting is to make the solution possible for the two sets of rotation parameters. The inner zone,*fig (5)*, should be sufficiently close to the terrestrial initial point so that their coordinates in the second set will not contain significant systematic errors. The outer zone contains the remaining common network stations.

B) HOTINE MODEL

This model, *fig (6)* has two sets of rotation parameters, (R_x, R_y, R_z) for the geodetic system and $(\delta\alpha, \delta\beta)$ for the distortions in the terrestrial network. The azimuth change parameter $(\delta\alpha)$ is a rotation about the Z-axis of the local geodetic coordinate system at the terrestrial initial point (i). The zenith distance parameter $(\delta\beta)$ is a constant applied to all lines radiating from the terrestrial initial point (i). In addition this model has a scale difference parameter (Δ) to model the systematic scale error in the terrestrial network.

The expanded form of this model is;

$$F_k = \begin{bmatrix} Xo \\ Yo \\ Zo \end{bmatrix} + \begin{bmatrix} 1 & Rz & -Ry \\ -Rz & 1 & Rx \\ R_y & R_x & 1 \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} + \begin{bmatrix} 1+\Delta & -\delta\alpha & \cos\alpha_{ik}\,\delta\beta \\ \delta\alpha & 1+\Delta & \sin\alpha_{ik}\,\delta\beta \\ -\delta\beta/\cos\alpha_{ik} & 0 & 1+\Delta \end{bmatrix} \begin{bmatrix} X_{ik} \\ Y_{ik} \\ Z_{ik} \end{bmatrix} - \begin{bmatrix} X_k \\ Y_k \\ Z_k \end{bmatrix} = 0 \quad \ldots\ldots\ldots (5)$$

In which α_{ik} is the azimuth in the local geodetic coordinate radiating from the initial point (i). The estimation procedure is the same as *KRAKIWSKY-THOMSON MODEL*.

3- DATA UNDER CONSIDERATION

Twenty two Doppler stations were available up to the time of this study, *fig (7)*, the observations of these stations were made by three different groups as follows;

A) Tweleve stations were observed by the *512 specialist team Royal engineers (512 STRE)*, during the period from November 1977 to February 1978. These stations are *E7, T2, A5, A11, B14, B15, B19, B20, 2/50, N1001, W173 and X721*. The first eight stations are primary stations, while the other four are *2nd, 3rd and 4th* order stations. The coordinates of these stations are derived in terms of the *WGS 72* system from Doppler precise ephemeris observations. The estimated standard error of the Cartesian coordinates is 0.75 m in each axis [3].

B) six stations were observed throught the *ADOS project(African Doppler survey)*. The main purpose of this project was to establish a first order and well distributed geodetic network over the African contenent, determined from positioning techniques using Doppler satellite observations. These stations are *A4, B11, S12, A423, A360 and B10*. The coordinates were calculated by usuing Doppler precise ephemeris with a standard error of 0.75 m in each axis.

C) Four stations were observed by *ESA (Egyptian Survey Authority)*. These stations are *O1, A6, 2/C57, 2/C58*, and their coordinates are given with a standard error of 1.0 m in each axis.

It should be mentioned here that the standard errors given to the terrestrial coordinates were calculated according to a special iterative technique of weighting which makes $\Sigma V^T P V$ very close to unity [6]..

4- ESTIMATION PROCEDURES OF THE TRANSFORMATION PARAMETERS

Different least squares solutions were carrired out using all the models discribed before for two reasons;

A) Studying the main factors affecting each model. This includes the effect of number, distribution and quality of data points.

B) Performing a complete comparison between these different models for finding out the proper model to be used for the transformation between the geodetic and satellite coordinates in Egypt.

C) The statisticaly significant number of transformation parameters to be used in Egypt for each model.

Accordingly, the available data were splited into three groups. The purpose of such spliting is to study the effect of adding the 2nd, 3rd order points on the transformation parameters. The calssification of these groups are as follows;

GROUP A Consists of all the available doppler data points afterrejecting stations (*S12, B10, A360*).

GROUP B Consists of 16 Doppler points (*E7, O1, A6, 2/57, 2/58, 2/50, T2, A11, B11, A4, B14, B20, B15, X721, A5, B19*).

GROUP C Consists of 12 Doppler points of the first order network (*E7, O1, A6, T2, A11, B11, A4, B14, B15, A5, B19*).

4-1 ESTIMATION PROCEDURES FOR THE FIRST GROUP OF MODELS

The least squares estimation procedures for *Bursa, Molodensky & Veis Models* follows the following mathematical model;

$$F(L_a, X_a) = 0 \quad \text{or} \quad F(L_b + V, X_o + X) = 0 \qquad \ldots(6)$$

where

L_a	Adjusted observations	L_b	observed value
X_a	Adjusted parametres	X_o	Approximate parameters
V	Residuals	X	Unkown parameters

The usual least squares adjustment procedure $\Sigma\ V^T\ P\ V = \min$ subjected to the condition ;

$$A\,X + B\,V + W = 0 \qquad \ldots(7)$$

is applied , where

$$A = \partial F / \partial X_a \qquad B = \partial F / \partial L_a \qquad W = F(L_b, X_o)$$

The above model is usualy known in practice as the combined least square adjustment. The solution of the above system of equations (7) is given as

$$X = -(A^T H^{-1} A)^{-1} A^T H^{-1} W \qquad ...(8)$$

$$V = -P^{-1} B^T H^{-1} (A X + W) \qquad ...(9)$$

$$H = B P^{-1} B^T \qquad ...(10)$$

$$\sigma_o^2 = V^T P V / DF \qquad (11)$$

$$\Sigma_X = \sigma_o^2 Q_X \qquad (12)$$

$$Q_X = (A^T H^{-1} A)^{-1} \qquad (13)$$

4-2 ESTIMATION PROCEDURE FOR SECOND GROUP OF MODELS

The second group of models contains two sets of unknown parametrs, for this reason these groups require a special least-squares procedure. The estimation model [7] in functional form is given as;

$$F1(\bar{X}1, \bar{L}1) = 0 \qquad ......(14)$$

$$F2(\bar{X}1, \bar{X}2, \bar{L}2) = 0 \qquad ......(15)$$

where

$\bar{X}1$ transformation parameters (Xo ,Yo ,Zo ,Rx ,Ry ,Rz)

$\bar{X}2$ Scale difference and rotation parameters for the geodetic network.

$\bar{L}1$ The Observables $(X_{ik}, Y_{ik}, Z_{ik})^T_G$ and $(X_k, Y_k, Z_k)^T_D$ of the inner zone.

$\bar{L}2$ The Observables $(X_{ik}, Y_{ik}, Z_{ik})^T_G$ and $(X_k, Y_k, Z_k)^T_D$ of the outer zone

Linear Taylor series expansions of F1 and F2yields the following matrix equations

$$A_{11} X_1 + B_{11} V_1 + W_1 = 0 \qquad (16)$$

$$A_{21} X_1 + A_{22} X_2 + B_{22} V_2 + W_2 = 0 \qquad (17)$$

Where;

$A_{11} = \partial F1 / \partial \bar{X}_1$ $\qquad B_{11} = \partial F1 / \partial L$

$A_{21} = \partial F2 / \partial \bar{X}_1$ $\qquad A_{22} = \partial F2 / \partial \bar{X}_2$

$B_{22} = \partial F2 / \partial L$

The solution of the above least squares estimation model is simply given by using the following simple model

$$F (\bar{X}* + \bar{L}*) = 0$$

in which

$\bar{X}*$ = contains $\bar{X}1$ and $\bar{X}2$

$\bar{L}*$ = Contains $\bar{L}1$ and $\bar{L}2$

Accordingly the solution will follow the same procedures given in equations (8) to (13).

5-RESULTS AND RECOMMENDATIONS

Based on the sixty one least squares solutions made for our study as well as the analysis carried on it the following conclusions are summerised and drawn out;

1) Large discrebances were found at points *(S12, B10, A360)*, this indicates the existance of blunders either in the identification or in the geodetic coordinates.

2) Proper weights for each data point of different order should be applied when a combination between geodetic and Doppler coordinates is carried out.

3) For *MOLODENSKY MODEL*, it was found that 5 parameters only

(X_o ,Y_o ,Z_o , Δ, Ez) are equivalent to 7 parameters, but on the other hand for points of second and third order seven parameters must be used in the transformation .

4) The coordinates of the initial points can be considerd as the mean coordinates of all the points used in the transformation instead of *O1* or *F1*.

5) Points at distances < 300 km from the initial point have no effect on the scale factor and the rotational parameters and can be used only as a translation parameters.

6) For *VEIS MODEL*, six parameters (X_o ,Y_o ,Z_o , Δ ,$\delta\xi$,$\delta\eta$) are equivalent to seven parameters.

7) For the transformation from the *Egyptian* to the *Satellite datum* it is very useful to seperate the points according to its order, i.e transform the first order station seperately from the second and third order stations.

8) *KRAKIWSKY-THOMSON MODEL* and *HOTINE MODEL* are very sensitive to the number of points used in the transformation and also to the number of points considerd in the inner zone.

9) For *KRAKIWSKY-THOMSON MODEL* , some improvement was found in the transformation parameters and the residual values of the observations when;

a) All points at distances < 300 km from the initial point are considerd as points in the inner zone.

b) The coordinates of the initial point can be considerd as the mean coordinates of all points.

c) For stations of different orders take the points of the first order as points in the inner zone.

d) Seven parameters from this model (X_o ,Y_o ,Z_o ,Ey ,Δ ,$\delta\xi$,$\delta\eta$) are equivalent to ten parameters, when compared relative to $\Sigma V^T P V$ for each case.

10) For *HOTINE'S MODEL* it was found that

a) Although many trials were made to obtain the best case for this model by the elimination of some parameters, the translation parameters were the only significant parameters.

b) The elimination of the second set of rotation parameters have a minimum effect on the values of other parameters, but some improvements were noticed in the variances of this parameters.

Accordingly and based on the above conclusion the following recommendations can be outlined;

1) For the transformation from *Satellite to Egyptian system* and based on *Molodensky's model* ,the values of the following five parameters can be taken as the bestvalues ;

$$X_o = 125.00 \pm 0.63 \text{ m}$$

$$Y_o = -96.55 \pm 0.62 \text{ m}$$

$$Z_o = 19.68 \pm 0.65 \text{ m}$$

$$\Delta = -2.64 \pm 2.00 \text{ P.P.M.} \qquad W_z = -0.53'' \pm 0.62''$$

2) For points < 300 km from the initial point it is recommended to use only three parameters :

$$X_o = 124.83 \pm 0.64 \text{ m}$$

$$Y_o = -96.59 \pm 0.62 \text{ m}$$

$$Z_o = 19.93 \pm 0.63 \text{ m}$$

3) For the transformation from *Egyptian* to *Satellite system* the following parameters are used for first order station :

$$X_o = -124.65 \pm 0.61 \text{ m}$$

$$Y_o = 96.47 \pm 0.59 \text{ m}$$

$$Z_o = -20.07 \pm 0.63 \text{ m}$$

$$\Delta = 1.90 \pm 1.93 \text{ P.P.M.} \qquad W_z = 0.31'' \pm 0.61''$$

For second and third order stations the following parameters are used:

$$X_o = -124.24 \pm 1.25 \text{ m}$$

$Yo = 95.33 \pm 1.13$ m

$Zo = -19.55 \pm 1.15$ m

$\Delta = -1.30 \pm 3.98$ P.P.M. $Wx = 2.21'' \pm 1.39''$

$Wx = 1.61'' \pm 1.66''$ $Wz = 1.71'' \pm 0.97''$

4- It should be noted that the previous results can be improved by observing more Doppler stations, especially in the west desert and Sinai area.

5- At this stageit should be mentioned that GPS system is in prototype stage and will be operated in the very near future, concequently when sufficient GPS observations are done in Egypt simillar investigations to the contained herein must be done to achive a more accurate transformation parameters.

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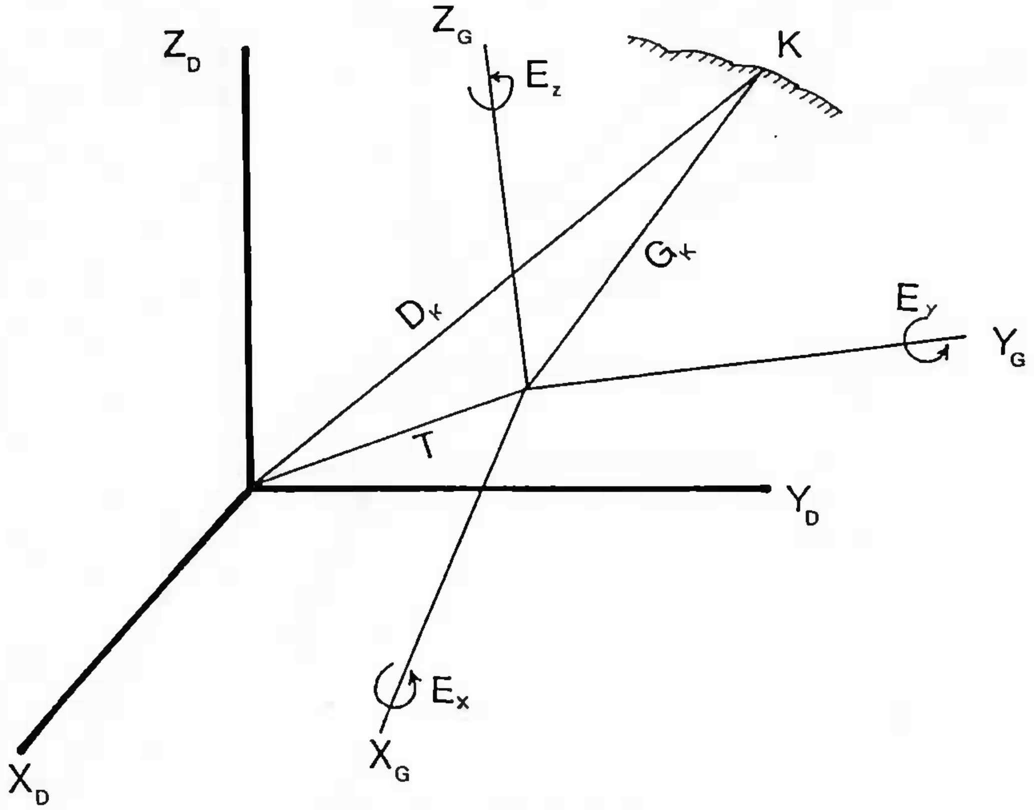

Fig. 1. Bursa Model

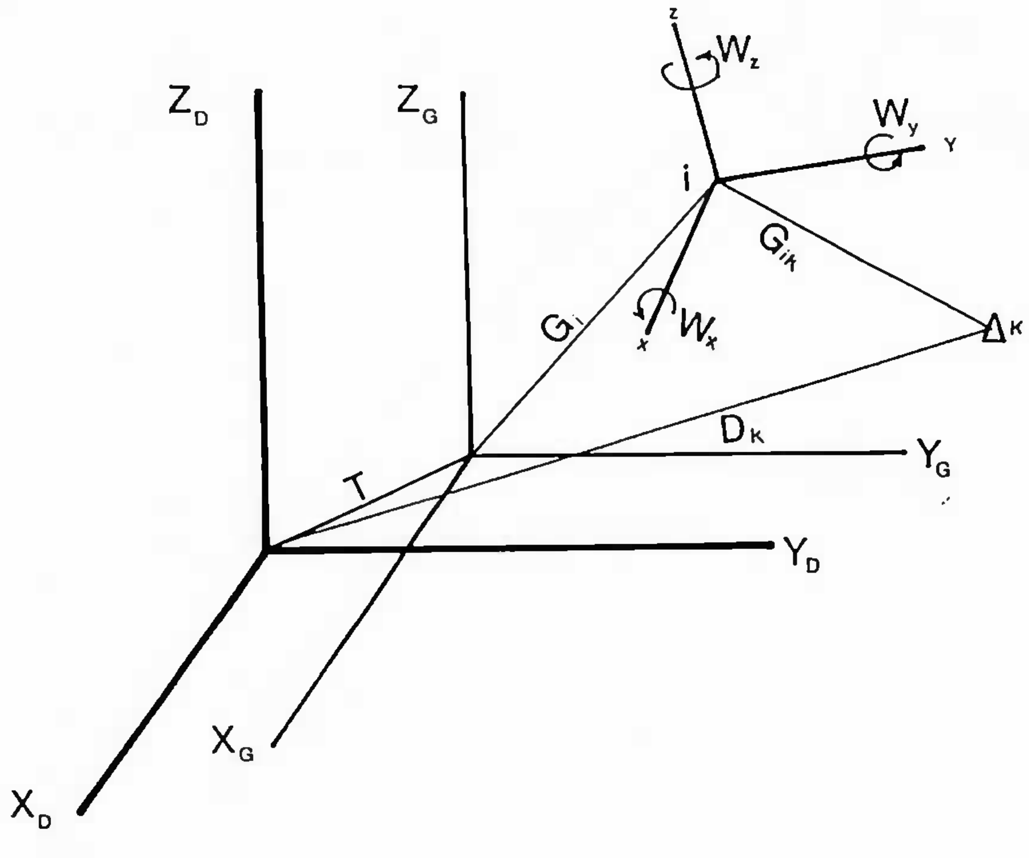

Fig. 2. Molodensky Model

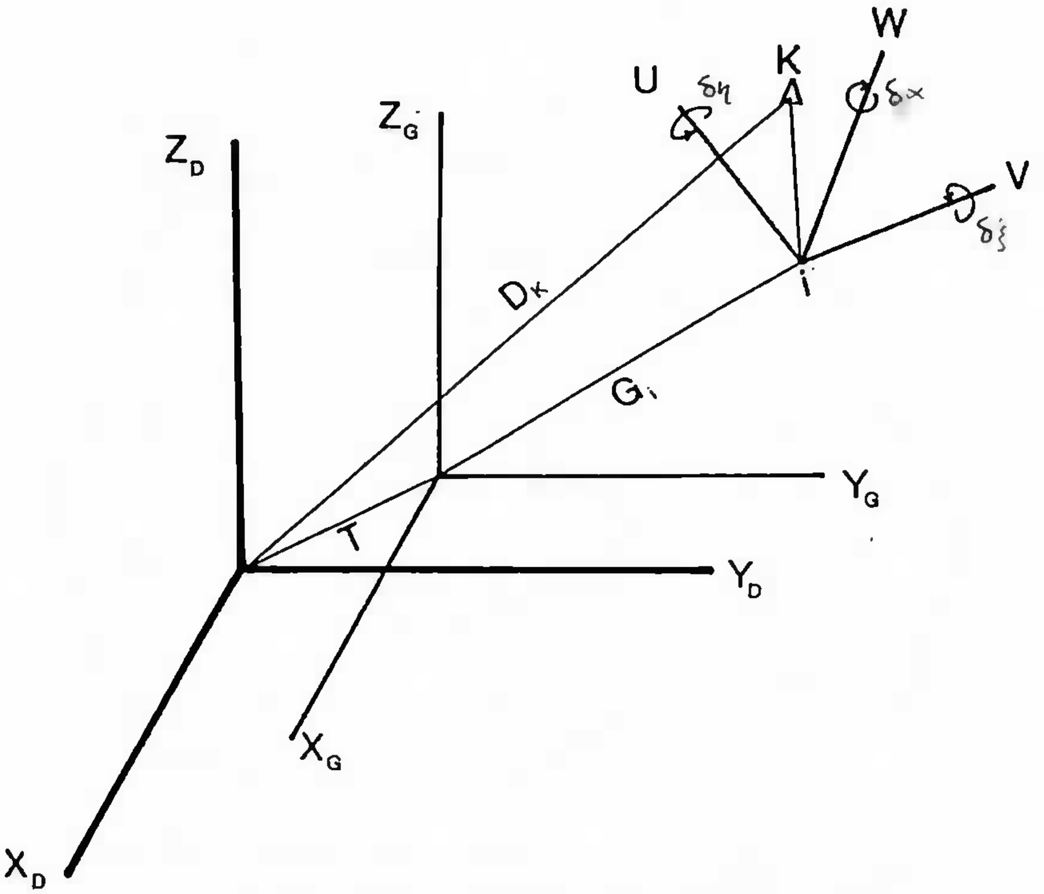

Fig. 3. Veis Model

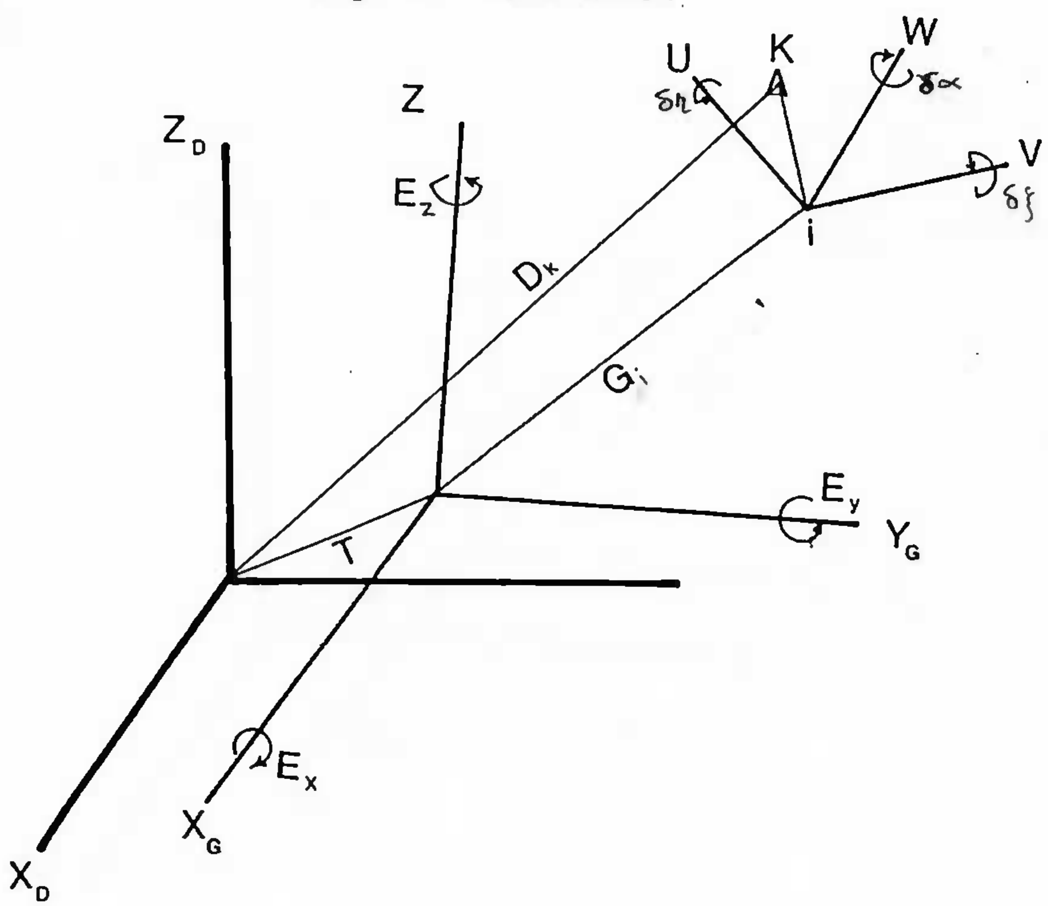

Fig. 4. Krakiwsky-Thomson Model

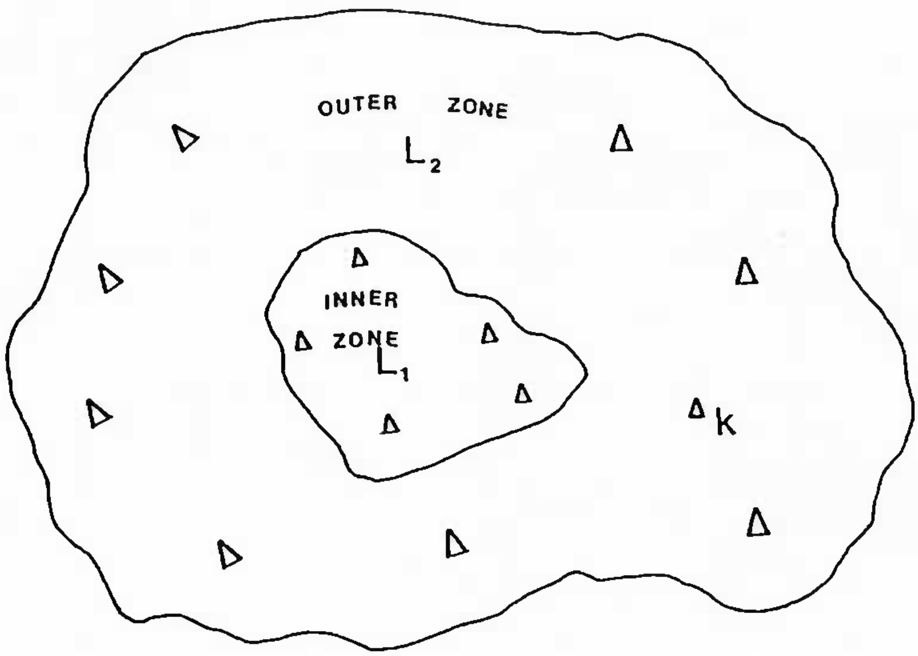

Fig. 5. Inner and Outer Zone Estimation for Krakiwsky-Thomson and Hotine Models.

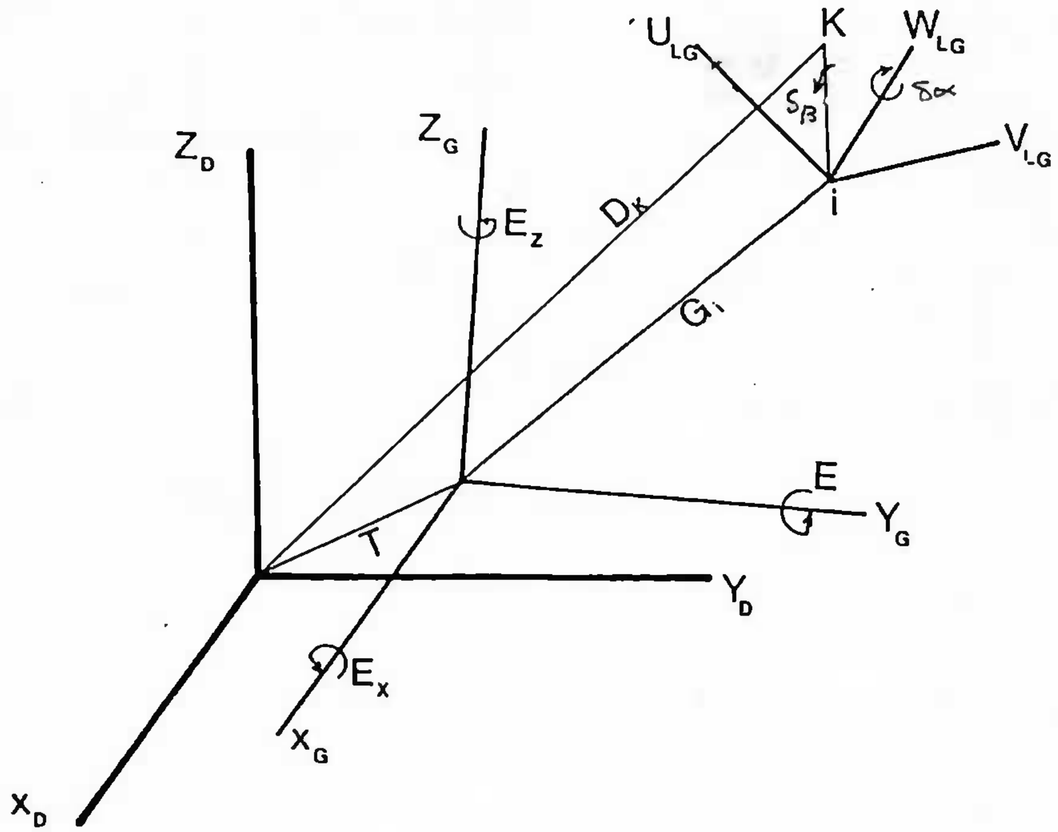

Fig. 6. Hotine Model

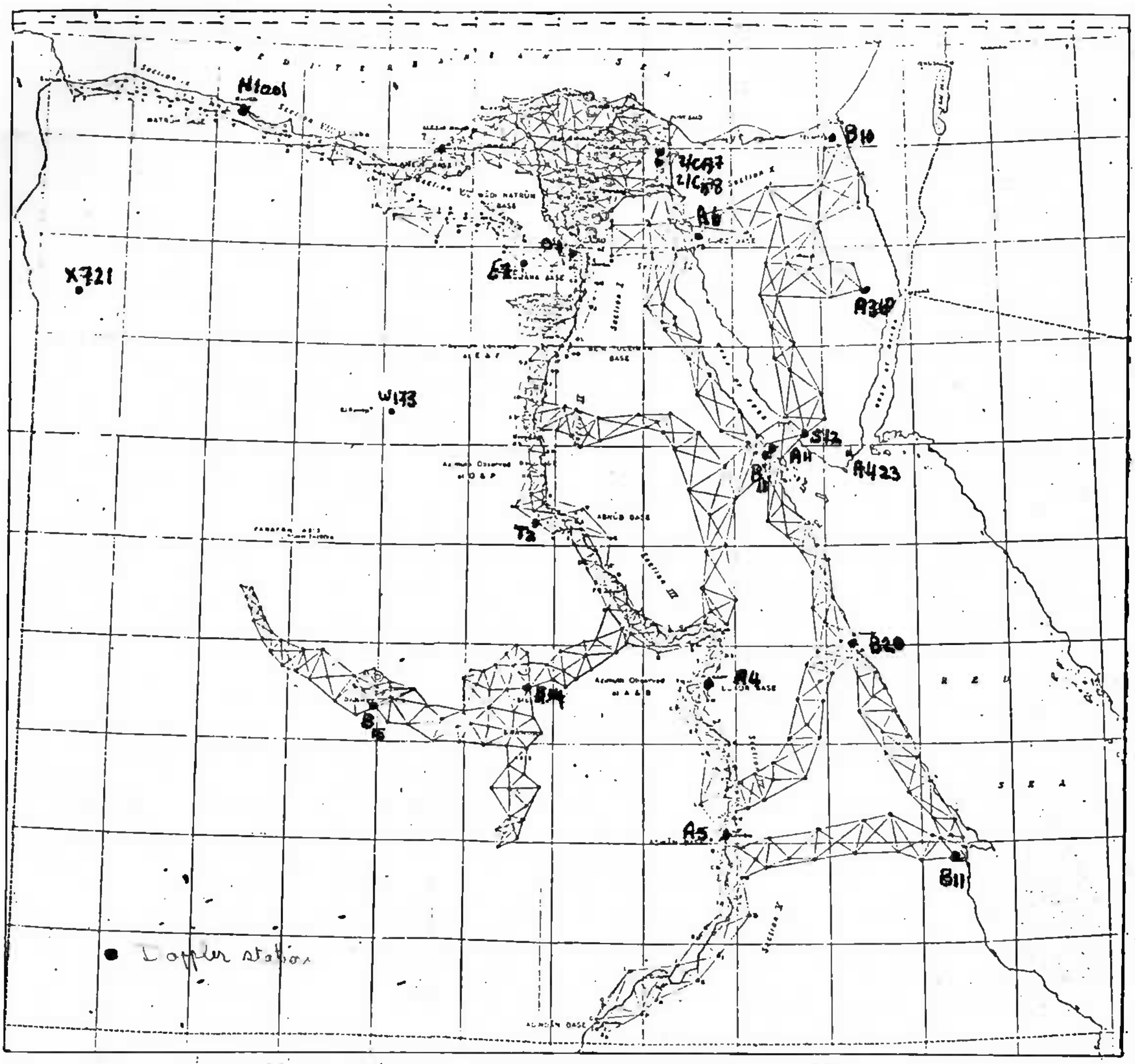

Fig. 7. Available Doppler Data Points